ELEMENTARY LINEAR ALGEBRA – SET 4

Analytic geometry in the 3d space

1. Find the values of the parameters t, s for which the vectors $\vec{v} = (1-t, 3, -1)$ and $\vec{u} = (-2, 4-s, 2)$ are parallel.

2. Find the values of the parameter t for which vectors $\vec{v} = (t, 2, 1-t)$ and $\vec{u} = (t, 1, -2)$ are perpendicular.

3. Compute the area of the parallelogram spanned by vectors $\vec{v} = (-1, 2, 5)$ and $\vec{u} = (0, 3, 2)$.

4. Compute the area of the triangle with vertices $A = (0, 1, 1)$, $B = (3, 0, 1)$ and $C = (0, 1, 2)$.

5. For the triangle in Problem 4 compute the length of the altitude through the vertex A.

6. Compute the volume of the parallelepiped spanned by vectors $\vec{v} = (1, 2, 3)$, $\vec{u} = (0, 4, 1)$ and $\vec{w} = (-1, 0, 2)$.

7. Compute the volume of the tetrahedron with vertices $A = (1, 1, 1)$, $B = (1, 2, 3)$, $C = (0, 4, 1)$ and $D = (2, 2, 2)$.

8. For the tetrahedron in Problem 7 compute the lenght of the altitude through the vertex A.

9. Find normal and parametric equations of the plane

   (a) through the points $P = (1, -1, 0)$, $Q = (2, 3, 7)$ and $C = (4, 0, 1)$.

   (b) through the point $P = (-2, 5, 4)$ and including the Oz axis,

   (c) through the point $P = (-1, 2, 4)$ and perpendicular to the Oy axis.

10. Do the parameteric equations

$$\begin{cases} x &= 3 - t + 2s \\ y &= -1 + t \\ z &= 2 + t - 3s \end{cases} \quad \text{and} \quad \begin{cases} x &= 4 + 3t + 3s \\ y &= t - s \\ z &= -2t - 4s \end{cases}$$

    describe the same plane? Justify your answer.

11. Find a parametric equation of the plane given by the equation $2x + y - z - 7 = 0$

12. Find a normal equation of the plane given by the parametric equation

$$\begin{cases} x &= t + s \\ y &= -2 - 2s \\ z &= 3 + 3t - s \end{cases}$$

13. Explain why the parametric equations

$$\begin{cases} x &= 1-t \\ y &= 2-3t \\ z &= 4t \end{cases} \quad \text{and} \quad \begin{cases} x &= 2t \\ y &= -1+6t \\ z &= 4-8t \end{cases}$$

describe the same line.

14. Find a parametric equation of the line in which two planes

$$\begin{cases} x+y-3 &= 0 \\ -y+z-1 &= 0 \end{cases}$$

intersect each other.

15. Find the intersection point of the line $l:\ x=t,\ y=1-2t,\ z=-3+2t$ and the plane $\pi:\ 3x-y-2z-5=0$.

16. Find the distance between the point $P=(1,0,2)$ and the plane $\pi:\ x+2y-3z+1=0$

17. Find the distance between the point $P=(2,5,1)$ and the line $l:\ x=t,\ y=1-2t,\ z=-3+2t$.

18. Find the distance between two parallel lines

$$\begin{cases} x+y+z-3 &= 0 \\ x-2y-z-1 &= 0 \end{cases} \quad \text{and} \quad \begin{cases} x+y+z-3 &= 0 \\ x-2y-z+4 &= 0 \end{cases}$$

Romuald Lenczewski
(most problems are taken from the lists of M. Gewert and Z. Skoczylas)